Surface behaviour of the pairing gap
in semi-infinite nuclear matter

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Abstract

The $^1S_0$-pairing gap in semi-infinite nuclear matter is evaluated microscopically using the effective pairing interaction recently found explicitly in the coordinate representation starting from the separable form of the Paris NN-potential. Instead of direct iterative solution of the gap equation, a new method proposed by V. A. Khodel, V. V. Khodel and J. W. Clark was used which simplifies the procedure significantly. The gap Δ obtained in our calculations exhibits a strong variation in the surface region with a pronounced maximum near the surface.

Recently [1, 2] a method was elaborated of explicit consideration of the gap equation for semi-infinite nuclear matter within an approach based on the concept of the effective pairing interaction. The use of a separable form of the NN-potential simplifies the problem significantly making it solvable numerically in the mixed coordinate-momentum representation. In Ref. [2] the effective pairing interaction in the $^1S_0$-channel was obtained explicitly without any form of local approximation for the separable 3×3 form [3, 4] of the Paris potential [5]. In this case the gap equation is reduced to a set of three integral equations in coordinate space with kernels which are expressed in terms of u- and v-functions obeying the Bogolyubov equations with nonlocal gap Δ. These equations have the integro-differential form that makes their solution rather complicate. In Ref. [6] a method was elaborated of solving such equations, but it proved out to be rather cumbersome. As it is well-known, a rather slow convergence is inherent to the usual iterative method of solution of the gap equation. Since each iteration requires to solve these integro-differential Bogolyubov equations, a huge cpu time is necessary to obtain a reliable solution in this way.

To make the procedure simpler, a new method [7] of solving the gap equation for the case of nonlocal interaction is used (we refer to it as KKC). This method was originally devised for infinite nuclear matter but it is valid for any situation provided the gap Δ is much less than the Fermi energy ε_{F}. The main computation problem of usual iterative scheme originates from a strongly nonlinear form of the integral gap equation with a lot of iterations necessary to obtain the solution. The main idea of the KKC method is that the nonlinear form is important only for the magnitude of the gap but not for its momentum dependence. As to the latter, it is determined by integrals over a wide momentum space which does not practically depend on Δ, provided the parameter $\Delta/\varepsilon_{\mathrm{F}}$ is small. The gap Δ was identically represented in [7] as a product $\Delta(k) = \Delta_{\mathrm{F}}\chi(k)$ of the constant $\Delta_{\mathrm{F}} = \Delta(k_{\mathrm{F}})$ and the "gap-shape" function $\chi(k)$ normalized to $\chi(k_{\mathrm{F}}) = 1$. Then the initial nonlinear gap equation was changed by a set of two equations. The first one, for the gap-shape function $\chi(k)$, is a linear integral equation which does not practically depend on the value of Δ_{F} and can be readily solved. If the function $\chi(k)$ is known, the equation for the gap amplitude Δ_{F} is just an algebraic nonlinear one which can be solved straightforwardly with standard methods.

In the zero approximation of the KKC method, the gap-shape function $\chi^{(0)}(k)$ is calculated with $\Delta_{\mathrm{F}} = 0$. Then the zero approximation for the gap magnitude $\Delta_{\mathrm{F}}^{(0)}$ is readily found. At the next approximation, the value of $\Delta_{\mathrm{F}}^{(0)}$ is used as an input for calculating $\chi^{(1)}(k)$ and the procedure can be repeated. In principle, the KKC method contains no additional approximations in comparison with the standard one. It just rearranges the iterative scheme, making the convergence much faster. Already the zero approximation of the KKC method gives, as a rule, sufficiently

accurate results [7, 8].

The KKC method can be readily extended to the case of nonzero temperature T [7, 8]. In this case the gap-shape function is practically T-independent. The reason is that at small temperature which is of interest (the critical temperature is $T_c \simeq 0.5\Delta_F$) redistribution of particles occurs in very narrow region nearby the Fermi surface which does not practically influence $\chi(\mathbf{k})$. Especially simple result appears in the zero approximation of the KKC method:

$$\Delta(\mathbf{k}, T) = \Delta_F(T)\,\chi(\mathbf{k}). \tag{1}$$

In the semi-infinite matter an additional coordinate dependence appears, but all physical reasons of eq. (1)-type representation remain valid with the coordinate and momentum dependent, but T-independent, shape function χ and the only T-dependent factor $\Delta_F(T)$. The latter one can be found as the asymptotical value inside nuclear matter which coincides with that of infinite system and is obtained by a much more simple calculation. To find the shape function we go to the limit $T \to T_c$ where the gap equation becomes linear. Such a method is used in this paper. It turns out to be, in the case of semi-infinite nuclear matter, much more convenient than the direct solution of the gap equation.

In symbolic notation, the gap equation has the form [9, 10]:

$$\Delta(T) = \mathcal{V}_{\text{eff}}^{\text{p}}\, A_0^s(T)\, \Delta(T), \tag{2}$$

where $\mathcal{V}_{\text{eff}}^{\text{p}}$ is the effective pairing interaction acting in the model space in which the superfluid two-particle propagator A_0^s is defined.

In the zero KKC approximation the temperature dependence of the gap operator $\Delta(\mathbf{k}, T)$ in homogeneous nuclear matter can be separated from its momentum dependence in the form of eq. (1). The shape function $\chi(\mathbf{k})$ of the gap operator can be considered as a T-independent one up to the critical temperature T_c. If T is close to T_c, the gap Δ is negligible, and the propagator $A_0^s(T_c)$ in eq. (2) coincides with that of the nonsuperfluid system which will be denoted as $A_0(T)$. Hence at $T = T_c$ eq. (2) is reduced to a homogeneous linear integral equation for $\chi(\mathbf{k})$. Its eigenfunction corresponding to the eigenvalue $\lambda = 1$ is the shape-function we search for.

Let us consider now the gap operator for semi-infinite nuclear matter which is nonuniform in the x-direction. We assume that the temperature dependence can be separated from the shape factor in the form similar to eq. (1) :

$$\Delta(x_1, x_2, k_\perp^2, T) = \Delta_F(T)\,\chi(x_1, x_2, k_\perp^2), \tag{3}$$

where $\mathbf{k}_\perp$ denotes the momentum in the $\mathbf{s}$-direction which is perpendicular to the x-axis. We use here the effective interaction calculated in [2] for the separable form

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[3, 4] of the Paris potential with form factors $g_i(k^2)$ (i=1,2,3) depending on the relative momenta of interacting particles. In notations of [2], it has the form:

$$\mathcal{V}^{\mathrm{p}}_{\mathrm{eff}}(x_1, x_2, x_3, x_4; k_\perp^2, k_\perp'^2) = \sum_{ij} \Lambda_{ij}(X, X')\, g_i(k_\perp^2, x)\, g_j(k_\perp'^2, x'), \qquad (4)$$

where $X=(x_1+x_2)/2$, $X'=(x_3+x_4)/2$, $x=x_1-x_2$, $x'=x_3-x_4$, and $g_i(k_\perp^2, x)$ stands for the inverse Fourier transform of the form factor $g_i(k_\perp^2 + k_x^2)$ in the x-direction. It is obvious that the gap-shape factor can be also written as a sum

$$\chi(x_1, x_2; k_\perp^2) = \sum_i \chi_i(X)\, g_i(k_\perp^2, x). \qquad (5)$$

After substitution of eqs. (3)–(5) into eq. (2) at $T=T_c$ we arrive at the following equation for the components χ_i:

$$\chi_i(X) = \sum_{lm} \int dX_1 dX_2\, \Lambda_{il}(X, X_1)\, B^0_{lm}(X_1, X_2, T_c)\, \chi_m(X_2), \qquad (6)$$

where

$$B^0_{lm}(X_1, X_2, T) = \int \frac{d\mathbf{k}_\perp}{(2\pi)^2}\, dx_1\, dx_2\, g_l^*(k_\perp^2, x_1)\, g_m(k_\perp^2, x_2) \times$$

$$\times A_0\left(X_1 + \frac{x_1}{2}, X_1 - \frac{x_1}{2}, X_2 + \frac{x_2}{2}, X_2 - \frac{x_2}{2}; k_\perp^2, T\right). \qquad (7)$$

As is known [11], for the two-particle propagator $A_0(T)$ at $T > 0$ the Matsubara technique leads to the same expression as at $T=0$:

$$A_0(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4, E = 2\mu; T) = \sum_{\lambda, \lambda'} \frac{1 - N_\lambda - N_{\lambda'}}{2\mu - \varepsilon_\lambda - \varepsilon_{\lambda'}}\, \varphi_\lambda(\mathbf{r}_1)\varphi_\lambda^*(\mathbf{r}_3)\, \varphi_{\lambda'}(\mathbf{r}_2)\varphi_{\lambda'}^*(\mathbf{r}_4), \qquad (8)$$

with T-dependent occupation numbers

$$N_\lambda(T) = \frac{1}{1 + e^{(\varepsilon_\lambda - \mu)/T}}, \qquad (9)$$

where μ is the chemical potential of the system. The summation in eq. (8) is carried out over the single-particle states $\lambda = \{n, \mathbf{k}_\perp\}$ with functions $\varphi_\lambda(\mathbf{r}) = y_n(x)\, e^{i\mathbf{k}_\perp \mathbf{s}}$ and energies $\varepsilon_\lambda = \varepsilon_n + k_\perp^2/2m$.

After simple transformations we finally obtain

$$B^0_{lm}(X, X', T) = -\sum_{n_1 n_2} \int \frac{d\mathbf{k}_\perp}{(2\pi)^2}\, \frac{1 - N_{n_1}(k_\perp, T) - N_{n_2}(k_\perp, T)}{\varepsilon_{n_1} + \varepsilon_{n_2} + k_\perp^2/m - 2\mu} \times$$

$$\times g^l_{n_1 n_2}(k_\perp^2, X)\, g^m_{n_1 n_2}(k_\perp^2, X'), \qquad (10)$$

where

$$g_{n_1 n_2}^l(k_\perp^2, X) = \int g_l(k_\perp^2, x)\, y_{n_1}(X + \frac{x}{2})\, y_{n_2}(X - \frac{x}{2})\, dx. \tag{11}$$

The effective interaction $\mathcal{V}_{\mathrm{eff}}^{\mathrm{p}}$ [2] is defined in such a way that the model space involves only the single-particle states with negative energies. Therefore the summation over n_1, n_2 and integration over $\mathbf{k}_\perp$ in eq. (10) is limited by the condition $\varepsilon_{\lambda_1}, \varepsilon_{\lambda_2} < 0$.

The above formulae determine the kernels of eq. (6) for the gap-shape function $\chi(x_1, x_2, k_\perp^2)$. Solution of this equation should be substituted into eq. (3) together with the factor of $\Delta_{\mathrm{F}}(T){=}0$ which can be found at $X_{12}{\to}{-}\infty$. Therefore it coincides with that of infinite nuclear matter.

Before going over to solution of the above equations for the case of semi-infinite nuclear matter, let us check the accuracy of eq. (1), i.e. the zero approximation of the KKC method, in infinite matter for the specific kind of the NN-potential in the separable form [3, 4]. It was used for studying pairing in neutron and nuclear matter within the Brueckner theory in Refs. [12, 13]. We present here some results of these calculations in the form which is convenient for analysis of the T-dependence of the gap-form function. For the separable potential under consideration the momentum dependence of Δ is given by coefficients C_i of the sum

$$\Delta(k, T) = \sum_i C_i(T)\, g_i(k^2). \tag{12}$$

In fact, it is sufficient to check the T-independence of the ratios of these coefficients to each other. It results that the T-independence of the ratios of C_2/C_1 and C_3/C_1 is really true for both neutron and nuclear matter for different densities up to $T = T_c$ within $2\%{\div}3\%$- accuracy. An example for nuclear matter at $k_{\mathrm{F}} = 0.8$ fm^{-1} which corresponds to the maximum value of Δ is given in Table 1.

Let us return to consideration of semi-infinite matter. Eq. (6) can be considered as a particular case of a more general set of homogeneous integral equations

$$\chi_i(X) = \lambda(T) \sum_j \int dX'\, K_{ij}(X, X'; T)\, \chi_j(X), \tag{13}$$

where $K_{ij}(X, X'; T)$ are the kernels of this equation for arbitrary T and $\lambda(T)$ stands for the eigenvalue. The critical temperature can be found from the condition that the minimum eigenvalue $\lambda_1(T_c)$ is equal to unit. In principle, such a way which involves solution of eq. (13) for a number of values of T is possible but rather cumbersome. In the situation under consideration, when the pairing in infinite nuclear matter exists, it is obvious that T_c in semi-infinite matter is the same as in infinite one. Therefore it is much simpler to find T_c for infinite matter and use it in eq. (6) (or eq. (13) at $\lambda{=}1$, $T{=}T_c$) as an input.

Table 1

Demonstration of the T-independence of the gap-shape
function for nuclear matter at $k_F = 0.8$ fm^{-1}.

T, MeV	C_2/C_1	C_3/C_1
0	0.355	-0.0271
0.25	0.355	-0.0271
0.50	0.355	-0.0271
0.75	0.354	-0.0271
1.00	0.353	-0.0270
1.25	0.350	-0.0270
1.30	0.349	-0.0269
1.35	0.349	-0.0269
1.40	0.348	-0.0269
1.45	0.347	-0.0269
1.50	0.347	-0.0270
$1.55(\simeq T_c)$	0.348	-0.0271

Let us now describe briefly how to calculate the set of integral equations, eq.(6). The kernels $K_{im}(X, X')$ of these equations are given by folding of the coefficients $\Lambda_{il}(X, X_1)$ of the effective interaction $\mathcal{V}_{\text{eff}}^p$ with those $B_{lm}^0(X_1, X'; T_c)$ of the propagator $A_0(T{=}T_c)$. The first ones were calculated in [2] for the value of chemical potential $\mu = -8$ MeV which simulates the situation in finite nuclei. In this case the critical temperature is $T_c{=}0.124$ MeV. In fact, the interval $\{X\} = (-8\,\text{fm}, 8\,\text{fm})$ was considered only because properties of $\mathcal{V}_{\text{eff}}^p$ are trivial outside this interval. So, at $X, X' < -4$ fm the magnitude of $\mathcal{V}_{\text{eff}}^p$ coincides practically with that calculated for infinite matter, whereas at $X, X' > 4$ fm it tends rapidly to the free T-matrix taken at negative two-particle energy $E = 2\mu$. As it is shown in [2], at a fixed value of $X_0{=}(X{+}X')/2$ the effective interaction $\mathcal{V}_{\text{eff}}^p(X, X')$ vanishes rapidly when the relative distance $t{=}X{-}X'$ is growing, so that integrals involving $\mathcal{V}_{\text{eff}}^p(X, X')$ can be cut at $|t| > 4$ fm. That is why the interval $(-8\,\text{fm}, 8\,\text{fm})$ for the effective interaction is sufficient for any calculations. As to eq.(6) for the gap-form function, a much wider X-space should be considered, $\{X\} = (-L_{\text{in}}, L_{\text{ex}})$ with a minimum value of $L_{\text{in}} \simeq 40$ fm (the value of $L_{\text{ex}} = 8$ fm is large enough). The reason is that at small value of $\Delta_F \lesssim 1$ MeV the correlation length $\xi \sim v_F/\Delta_F$ is very big ($\xi > 10$ fm). The distance between the left cut-off L_{in} and the point under consideration (say, $X \gtrsim -10$ fm) should exceed a correlation length for the effects of the left boundary not to distort the asymptotic behaviour of the gap-shape function inside nuclear matter essential-

ly. For $X, X' < -8$ fm we use the effective interaction $\mathcal{V}_{\text{eff}}^{\infty}(t)$ calculated for infinite nuclear matter. It is obvious that it depends only on the difference $t = X - X'$.

The propagators $B_{lm}^{0}(T)$ are new ingredients of the problem. They can be readily calculated in accordance with formulae (10), (11) using the technique elaborated in [2].

Such a large X-space makes it difficult to solve eq. (6) in the coordinate space directly along the way which was used in Ref. [2] for a similar equation for $\mathcal{V}_{\text{eff}}^{\text{p}}$. It is more convenient to use the Fourier expansion of the gap-shape function in the interval $(-L_{\text{in}}, L_{\text{ex}})$:

$$\chi_i(X) = \sum_n \chi_i^n f_n(X) \tag{14}$$

where $f_n(x)$ are $\sin(2\pi n(X - X_c)/L)$ and $\cos(2\pi n(X - X_c)/L)$, $L = L_{in} + L_{ex}$, $X_c = (L_{ex} - L_{in})/2)$. The kernels $K_{ij}(X, X')$ are also expanded in the double Fourier series. Finally, we deal with a set of homogeneous linear equations for the coefficients χ_i^n :

$$\chi_i^n = \sum_{j=1}^{3} \sum_{n'=1}^{N} K_{ij}^{nn'} \chi_j^{n'}, \tag{15}$$

with the matrix $K_{ij}^{nn'}$ having not very high dimension. In this calculation we used the interval of $\{X\} = (-40\,\text{fm}, 10\,\text{fm})$ $(X_c = -15\,\text{fm})$, the value of $N = 101$ being enough to obtain the accuracy better than 1%. When the coefficients χ_i^n are found, the gap-shape function $\chi(X, t, k_\perp^2)$ can be readily obtained with use of eqs. (14) and (5). To obtain the complete gap at zero temperature $\Delta(X, t, k_\perp^2, T = 0)$, one has just to multiply χ by a constant $\Delta_{\text{F}} = 0.167$ MeV found for infinite nuclear matter.

To present the results in a more transparent way, it is convenient to calculate the zero moment of $\chi(X, x, \mathbf{s})$ over the relative coordinates:

$$\chi_0(X) = \int d\mathbf{s} \int dt \, \chi(X + t/2, X - t/2, \mathbf{s}) = \sum_i \chi_i(X). \tag{16}$$

Here, in accordance with Ref. [2], the normalization $g(k^2 = 0) = 1$ was used. Let us also consider the function

$$\chi_{\text{F}}(X) = \sum_i \chi_i(X) \, g_i(k^2 = k_{\text{F}}^2(X)), \tag{17}$$

where $k_{\text{F}}(X) = \sqrt{2m(\mu - U(X))}$ is the local Fermi momentum ($k_{\text{F}}(X) = 0$ for $\mu - U(X) < 0$). The corresponding moments of the gap are as follows: $\Delta_0(X) = \Delta_{\text{F}} \chi_0(X)$, $\Delta_{\text{F}}(X) = \Delta_{\text{F}} \chi_{\text{F}}(X)$. The latter function is very important for the interpretation of results. It gives approximately matrix elements of Δ for the single-particle states taken at the Fermi surface. Analysis of this approximation for the case of the semi-infinite geometry is given in Ref. [2]. It should be noted that this

type of the local approximation is commonly used for description of pairing in finite nuclei [14]. Its accuracy for neutron stars was examined in [15]. It worth to stress that in our paper it is used only for a more transparent presentation of results which were obtained without this approximation.

Similar calculations were repeated for the chemical potential $\mu = -16$ MeV which is the "self-consistent" value for semi-infinite nuclear matter coinciding with that of infinite matter. In this case we have $T_c = 0.57$ MeV and $\Delta_{\mathrm{F}} = 0.94$ MeV.

Both functions, $\Delta_0(X)$ and $\Delta_{\mathrm{F}}(X)$, are drawn in Fig.1 for both values of μ. As it is seen, all the curves possess pronounced surface maxima. Such a surface behaviour appears due to a big surface value of the effective pairing interaction [2].

For more convenient analysis of the μ-dependence of the surface effect, the gap-shape functions $\chi_{\mathrm{F}}(X)$ for both values of μ are drawn together in Fig.2. As it is seen, the surface effect in Δ is more pronounced for $\mu = -8$ MeV. Indeed, in this case the ratio of the maximum surface value to the asymptotic one inside the matter is $\simeq 1.8$, whereas it is $\simeq 1.5$ for $\mu = -16$ MeV. Besides that, the position of the maximum is closer to the surface ($X = 0$) for $\mu = -8$ MeV than for $\mu = -16$ MeV.

It should be noted that the surface effect for Δ turns out to be less pronounced than the one for the effective interaction itself. This is a consequence of a strong surface-volume coupling which takes place in the gap-equation due to a big pairing correlation length which is significantly larger than the width of the surface layer. Such a coupling is inherent to a pure quantum calculation and can be partially lost when a local approximation is used. It suppresses partially the surface maximum in Δ and makes the μ-dependence of the surface effect smoother.

Such a dependence of the gap on μ originates from two reasons. The first one is a direct energy dependence of the effective interaction $\mathcal{V}_{\mathrm{eff}}^{\mathrm{p}}$ taken at $E = 2\mu$ [2]. The second one is the strong momentum dependence of the form-factors $g_i(k^2)$ taken at $k^2 = k_{\mathrm{F}}^2(X) = 2m(\mu - U(X))$. Both effects cooperate in enhancing the surface values of the effective interaction and of Δ for smaller values of μ. However, there is an effect which works in the opposite direction making the μ-dependence of the surface maximum of Δ less pronounced. This is a decrease of Δ_{F} at smaller values of $|\mu|$ which results in growing the correlation length. The latter smooths the surface effect itself and its μ-dependence.

The energy dependence of pairing should be important for consideration of the nuclear drip-line where the chemical potential μ tends to zero. The analysis of situation at small values of μ is in progress.

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Figure captions.

Fig. 1. The zero moment $\Delta_0(X)$ of the gap (dashed lines), and the function $\Delta_{\mathrm{F}}(X)$ (solid lines) for two values of μ.

Fig. 2. The gap-shape $\chi_{\mathrm{F}}(X)$ for $\mu=-8\mathrm{MeV}$ (dashed lines), and for $\mu=-16\mathrm{MeV}$ (solid lines).
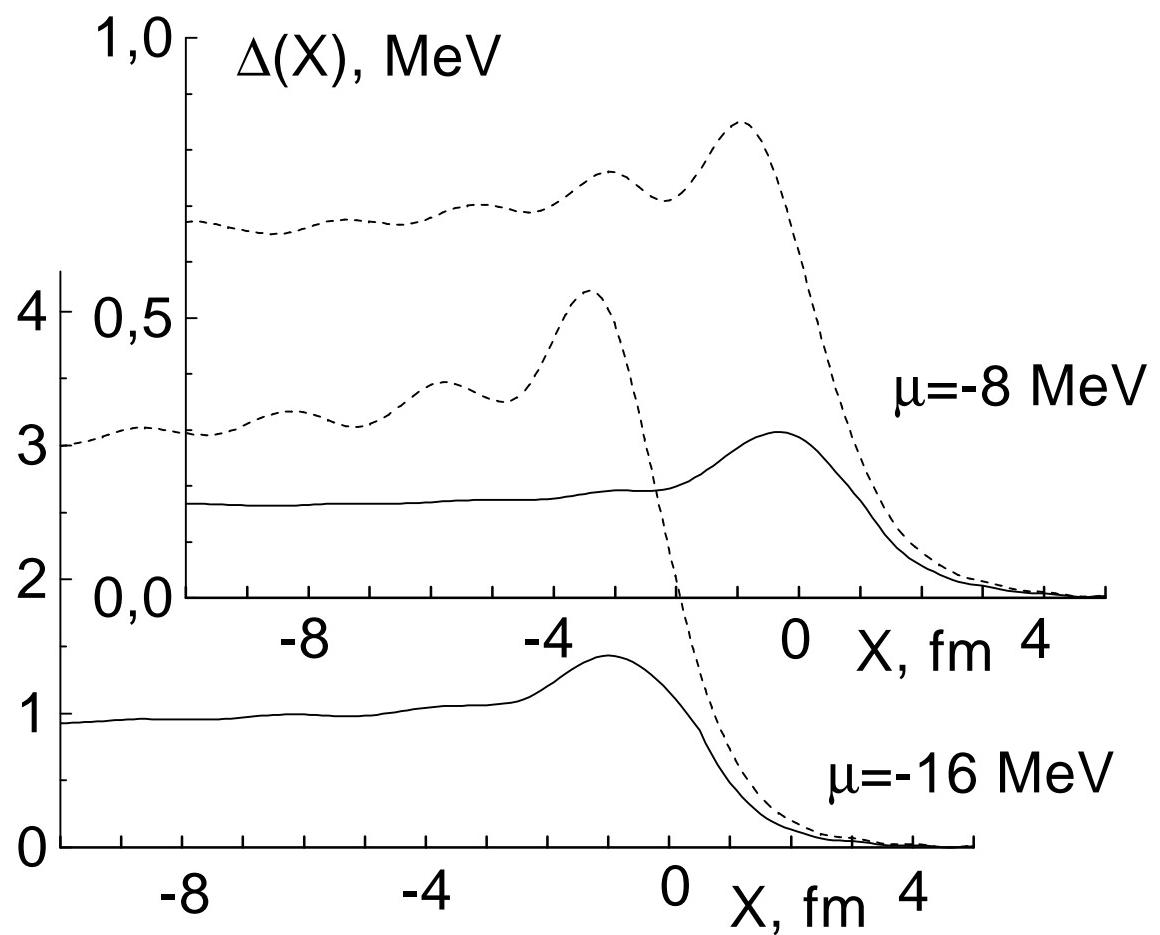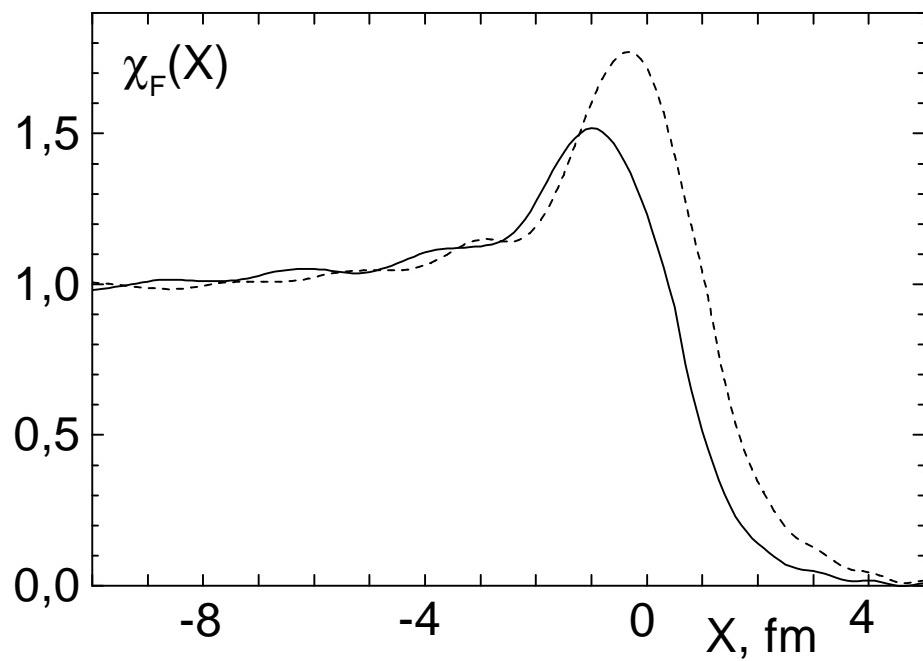
Fig. 1

Fig. 2